SMPTE RECOMMENDED PRACTICE

Derivation of Basic Television Color Equations

1 Scope

This practice is intended to define the numerical procedures for deriving basic color equations for color television and other systems using additive display devices. These equations are first, the normalized reference primary matrix which defines the relationship between RGB signals and CIE tristimulus values XYZ; then, the system luminance equation; and finally, the color primary transformation matrix for transforming signals from one set of reference primaries to another set of reference primaries or to a set of display primaries.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this practice. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this practice are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below.

CIE Publication 15.2 (1986), Colorimetry

3 Reference primary matrix

3.1 Input data

3.1.1 For television systems, reference white for both reference camera and display is CIE illuminant D_{65}. The chromaticities of D_{65}, rounded to four significant digits (CIE 15.2), are:

$x = 0.3127$
$y = 0.3290$

Other displays may utilize other white points. The CIE coordinates of some other standard CIE illuminants are:

	x	y
D_{55}	0.3324	0.3474
D_{50}	0.3457	0.3585
Ill C	0.3101	0.3162

(A more complete discussion of reference white and operating white is given in annex A.)

3.1.2 The chromaticity coordinates of the system reference primaries or display primaries shall be specified to a minimum of three significant digits using the 1931 CIE system of colorimetry (x, y coordinates). More precision may be used in the starting data if available. If the primary chromaticities are provided in the CIE 1976 uniform chromaticity coordinate systems (u′,v′), then they must be transformed to the 1931 CIE x, y chromaticity coordinates using the transformation given below (see annex D (1)).

$$x = \frac{9u'}{12 + 6u' - 16v'}$$

$$y = \frac{4v'}{12 + 6u' - 16v'}$$

Should the primary chromaticities be provided in the obsolete 1960 UCS system (u,v), these values should first be transformed to the 1976 system and then to the 1931 x,y values.

$$u' = u; \quad v' = 1.5\,v.$$

3.1.3 In the event that starting data are not available as chromaticity coordinates for a white

Approved November 1,1993

point or display primaries, they should be computed from the spectral power distributions using the colorimetric integration tables and procedures given in CIE 15.2.

3.2 Output data

The numerical coefficients of the output matrix must be computed accurately to four decimal digits. To avoid effects of rounding and truncation errors and to ensure this accuracy in the final result, it is recommended that all computations be carried out to 10-digit accuracy with the output matrix being derived directly from the chromaticity coordinates of the RGB primaries and reference white with no rounding or truncation of intermediate results. The final 10-digit result is rounded to the required 4-digit accuracy. A set of example computations accurate to 10 digits is given in annex B to aid in checking computational equipment.

3.3 General procedure

The general procedure for deriving the matrix relating normalized linear RGB signals to CIE XYZ tristimulus values is described in this clause and an example derivation is given in annex B. The RGB signals are normalized such that reference white has the values R=G=B=1.0. The step-by-step process is as follows:

3.3.1 Obtain the CIE x y chromaticity coordinates of the reference white (D_{65} for television) and of the RGB primaries.

3.3.2 Compute the z coordinate for the reference white and each of the RGB primaries:

$$z = 1 - (x + y)$$

3.3.3 Form the following matrix and column vector from the x y z numerical values of the reference primaries and white:

$$P = \begin{bmatrix} x_R & x_G & x_B \\ y_R & y_G & y_B \\ z_R & z_G & z_B \end{bmatrix} \qquad W = \begin{bmatrix} x_W / y_W \\ 1 \\ z_W / y_W \end{bmatrix}$$

Note that the W vector, representing the reference white, has been normalized so that white has a luminance factor of 1.0; i.e., Y = 1.0. This is necessary so as to cause the video reference white signal (R=G=B=1) to produce the reference white with a unity luminance factor.

3.3.4 Compute the coefficients C_i on the left side of the equation below by multiplying the W vector by the inverse of the P matrix. Note the notation P^{-1} indicates the matrix inversion operation. These coefficients are normalization factors which normalize the units of the RGB primaries such that a unit amount of each combine to produce the white point chromaticities with a luminance factor of 1:

$$\begin{bmatrix} C_R \\ C_G \\ C_B \end{bmatrix} = P^{-1} \bullet W$$

3.3.5 Form the diagonal matrix from the coefficients C_i computed in 3.3.4:

$$C = \begin{bmatrix} C_R & 0 & 0 \\ 0 & C_G & 0 \\ 0 & 0 & C_B \end{bmatrix}$$

3.3.6 Compute the final normalized primary matrix NPM as the product of the P and C matrices:

$$NPM = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} = P \bullet C$$

3.3.7 This matrix, NPM, is the final result and relates television linear RGB signals to CIE XYZ tristimulus values as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \bullet \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

3.3.8 The luminance equation for this set of primaries is the second row of the NPM matrix:

$$Y = Y_R (R) + Y_G (G) + Y_B (B)$$

In some cases, the NPM matrix values rounded to four digits may result in a luminance equation whose terms do not sum to 1.0. In that situation, the NPM matrix should be column normalized to force the second row to sum to 1.0.

3.3.9 Computations of color-difference signal coefficients should use all 10 digits of the luminance equation as determined above. These data should be multiplied by applicable scaling factors before rounding. Round to four decimal places and/or four digits, whichever extends the number further.

In some cases, the coefficients of the color-difference equations may not sum to zero after rounding. In that situation, the coefficients should be renormalized to force the coefficients of each equation to sum to zero.

4 Transformation between primary sets

4.1 Input data

The input data consists of the normalized primary matrices for a source system (NPM_S) and for a destination system (NPM_D). Ideally these matrices should have been generated directly from the 3-digit primary chromaticities and 4-digit reference white and, therefore, be of 10-digit accuracy. However, normalized reference primary matrices rounded to four digits may be used. It must be recognized that this will result in a lower precision transformation (see annex D (2)).

4.2 General procedure

4.2.1 Given the normalized primary matrices for the source (NPM_S) and destination (NPM_D) systems, the following equations relate CIE tristimulus values to the linear RGB signals in both source and destination systems:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = NPM_S \bullet \begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix} \text{ and } \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = NPM_D \bullet \begin{bmatrix} R_D \\ G_D \\ B_D \end{bmatrix}$$

The inverse relationships, predicting RGB from XYZ, may also be written:

$$\begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix} = NPM_S^{\,-1} \bullet \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \text{ and } \begin{bmatrix} R_D \\ G_D \\ B_D \end{bmatrix} = NPM_D^{\,-1} \bullet \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Again, the (-1) notation of the NPM matrices indicates matrix inversion.

4.2.2 The objective is to determine a matrix which transforms RGB signals from the source system into appropriate signals for the destination system. Start from the source, using its NPM to predict XYZ values from source RGB signal values as shown in the left equation below. Then write the equation predicting the destination RGB signals from XYZ values as shown in the right equation below:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = NPM_S \bullet \begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix} \text{ and } \begin{bmatrix} R_D \\ G_D \\ B_D \end{bmatrix} = NPM_D^{\,-1} \bullet \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Since the values of XYZ should be the same for both source and destination systems, the XYZ vector on the right side of the right equation can be replaced with the entire right side of the left equation:

$$\begin{bmatrix} R_D \\ G_D \\ B_D \end{bmatrix} = NPM_D^{\,-1} \bullet NPM_S \bullet \begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix}$$

4.2.3 The desired transformation matrix TRA is the product of NPM_D inverse and NPM_S:

$$TRA = NPM_D^{\,-1} \bullet NPM_S$$

and

$$\begin{bmatrix} R_D \\ G_D \\ B_D \end{bmatrix} = TRA \bullet \begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix}$$

Annex A (informative)
Reference white and RGB signal values

The role of the reference white, in this practice, is simply that of normalizing the units of the R G B primaries. That is, the relative video signal levels R=G=B=1 correspond to the reference white in the scene or on the display. All current television systems specify CIE illuminant D_{65} for both source and display. The signal processing on television signal sources is based on this white-point assumption. In practice, television cameras are used to produce images under a wide range of lighting conditions ranging from tungsten to very high color temperatures, and CIE illuminant D_{65} is probably rarely encountered. The television camera is always white balanced so that a white object always produces R=G=B regardless of the color quality of the studio light. In effect, television systems have always tacitly assumed that the color reproduction goal is to reproduce all colors as though they had been illuminated by CIE illuminant D_{65}.

Annex B (informative)
Example derivation of normalized primary matrix

B.1 Given the reference white chromaticities:

$x = 0.3127$
$y = 0.3290$

and a set of reference primaries:

$x_R = 0.640$ $y_R = 0.330$
$x_G = 0.300$ $y_G = 0.600$
$x_B = 0.150$ $y_B = 0.060$

B.2 The following values of C_i are derived:

$C_R = 0.6443606239$
$C_G = 1.1919477979$
$C_B = 1.2032052560$

B.3 The values for the NPM matrix before rounding to four digits are:

$$NPM = \begin{bmatrix} 0.4123907993 & 0.3575843394 & 0.1804807884 \\ 0.2126390059 & 0.7151686788 & 0.0721923154 \\ 0.0193308187 & 0.1191947798 & 0.9505321522 \end{bmatrix}$$

B.4 The luminance equation is:

$$Y = 0.2126390059(R) + 0.7151686788(G) + 0.0721923154(B)$$

and rounded to four digits:

$Y = 0.2126(R) + 0.7152(G) + 0.0722(B)$, in which the coefficients sum to 1.0.

Annex C (informative)
Example derivation of primary transformation matrix

C.1 Given a source NPM:

$$NPM_S = \begin{bmatrix} 0.5671181859 & 0.1903210663 & 0.1930166748 \\ 0.2793268677 & 0.6434664624 & 0.0772066699 \\ 0.0000000000 & 0.0725032634 & 1.0165544874 \end{bmatrix}$$

and a destination NPM:

$$NPM_D = \begin{bmatrix} 0.4123907993 & 0.3575843394 & 0.1804807884 \\ 0.2126390059 & 0.7151686788 & 0.0721923154 \\ 0.0193308187 & 0.1191947798 & 0.9505321522 \end{bmatrix}$$

C.2 The resulting transformation matrix TRA before rounding is:

$$TRA_{S \to D} = \begin{bmatrix} 1.4085805665 & --.4085805667 & 0.0000000000 \\ --.0256675666 & 1.0256675666 & --.0000000001 \\ --.0254274151 & --.0440308720 & 1.0694582872 \end{bmatrix}$$

Annex D (informative)
Bibliography

1) Hunt, R.W.G. Measuring colour. London: Elis Horwood Limited; 1987.

2) Sproson, W.N. Colour science in television and display systems, p21. Bristol: Adam Hilger Ltd; 1983.